Integrating Islamic and Conventional Project Finance

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EXECUTIVE SUMMARY

This article analyzes the traditional modes of Islamic project financing in light of modern financial engineering. The vehicles elaborated include debt, hybrid, and equity instruments. The first category includes the Qardh Hasan (benevolent loan), Murabahah (mark-up), Ijara (leasing), and Islamic income/revenue bond facilities; the second incorporates the classical Mudharabah (profit-sharing) contract; while the third category comprises the Musharakah (equity) vehicle. The Mudharabah contract can be synthetically created as a combination of a Cost-plus (Murabahah) facility and an Option (Al-ikhtiyarat) instrument to endogenously determine the profit sharing parameter. It is malleable in two-dimensions depending on the
financial goals and the risk profile of the investor. Securitization of the Mudharabah facility avoids the controversy of Bai' al-dayn bi al-dayn (sale of one set of debt security with another). Finally, the design of financial contracts via variants of Mudharabah securities such as income bond and participating bond is illustrated in a numerical example. © 1999 John Wiley & Sons, Inc.

Islamic project finance contemplates the funding of any Islamically acceptable project by means of contracts which are in accordance with the Shari'ah. The project undertaken should convey economic benefits which exceed its economic cost (Brigham and Gapenski, 1991). The modes of financing selected should not only avoid riba, gharar, and maysir but also be economically efficient.¹ Economic growth and independence of the Muslim nations can only be feasible if the necessary Ijtihad (independent judgment in a legal or theological question based on the interpretation and application of the principles of Islamic jurisprudence) is used to harness the resources of its citizens to the maximum potential.²

Muslim societies in the Near East economically flourished in the medieval era due to the proliferation of several credit facilities and partnership vehicles (Udovitch, 1975).³ Cizakca (1996) vividly describes how the Muslims financed their maritime projects during the glorious Ottoman Empire with the profit sharing instrument of Mudharabah. Similarly, in thirteenth century Europe where fixed interest (usurious) contracts also faced religious restrictions, the King of England resorted to a production payment loan—a quasi-equity facility—for the development of Devon Silver Mines (Gimpel, 1976).⁴ Currently, Islamic financial engineering is being reinvigorated by many institutions such as Bank Islam Malaysia Berhad

¹ Riba is interpreted by Ibn Qayyim (n.d.) to imply (i) any form of unfair trade, market manipulation or engaging a market participant to trade under duress (riba al-fadl) and (ii) interest based debt contracts (riba al-nasi'ah). In the context of modern financial economics, riba al-nasi'ah can be defined as a real risk-free return from an investment vehicle or strategy. The element of gharar in a contract entails deception (Ibn Taymiya, n.d.; Thomas A. S., 1995). Promoting it preempts maysir which is gambling (qimar).

² According to Anwar (1995) these resources also include idle funds of Waqf (charitable endowments) and Zakat (Islamic wealth tax).

³The credit facilities according to Udovitch (1975) included deferred payment sales (Bai' Bit-ta'khir), advance payment (Bai' Salam), transfer of debt (Hawala), and draft (Saftaja). The partnership vehicles included the hybrid (Mudharabah) as well as the pure equity (Musharakah) instruments.

⁴Al-Qaradawi (1984) is of the view that riba has been forbidden not only by the Quran, but also by the scriptures of the Prophets preceding Muhammad (PBUH) such as the Old Testament and the Gospel according to Luke. Keen (1997) provides an excellent discourse of the change in the attitude of the Christian West toward Usury, from prohibition to acceptance.
and the Islamic Development Bank (Ismail, 1992a; Wilson and Khan, 1995).

The purpose of this article is to translate the basic modes of Islamic financing into conventional security design principles and then illustrate how these modes can be structured as a package to finance a complex project. In particular, the traditional Mudharabah security can be synthesized as a combination of a Murabahah facility and a Call Option. From this perspective, a methodology is proposed to derive the profit sharing ratio endogenously. Mudharabah is an extremely flexible instrument as it can be structured along two dimensions: (i) It can be financially engineered to satisfy the financial goals of its investor ranging from a pure income bond to a pure growth facility, and (ii) it can be molded to satisfy the risk preferences of its clientele. For instance, an extremely risk-averse agent in the economy may trade off part of the downside risk of the instrument for a lower profit sharing ratio. A Mudharabah facility can also be packaged for resale in the secondary market as it circumvents the controversy of Bai’al-dayn bi al-dayn (sale of one set of obligations, e.g., debt, with another—Kamali, 1996).

TRADITIONAL MODES OF ISLAMIC FINANCING

Islamic finance has developed a variety of instruments that may be respectively analyzed in conventional financial terms as debt, hybrid, and equity securities.

DEBT SECURITIES

Since exchanging money for more money (or monetary equivalents) over an extended period of time is interpreted as ribawi and hence forbidden in Islam, debt instruments have to be carefully structured so that the exchange involves goods for money or partnership shares for money over time. The intention is to facilitate trade or business and not to get around the religious injunction. It might be noted that conventional finance literature attributes ribawi loan transactions to the presence of (i) asymmetric information between insiders (managers) of firms and outside investors (Ross, 1977; Myers and Majluf, 1984), and (ii) conflict of interest (agency effect) between managers and shareholders of firms (Jensen and Meckling, 1976).\textsuperscript{5} If corporate

\textsuperscript{5}In the first instance debt serves as a signal to convey future prospects of a firm and in the second case debt precommits cash flow of a firm and reduces the chances of it being squandered by a manager in frivolous activities.
insiders were required to reveal vital information such as profitability and their stake in the firm on an ongoing basis and if managers were given performance incentives such as stock options, conventional debt instruments would lose much of their financial rationale in capital markets.

Debt instruments in Islam comprise the following: Qardh Hasan (Benevolent Loan), the Deferred Contracts of Bai’ Murabahah Bai’ Mua’jil (Cost Plus), Ijara (Leasing)/Ijara wal ‘Iqtina (Lease-purchase), and Islamic (Income/Revenue) Bonds.

Qardh Hasan involves a benevolent loan of funds (or fungible commodities) without any real interest (or excess in quality or quantity). It is the only risk-free asset allowed in Islam.\(^6\) In terms of Figure 1, a Qardh Hasan facility would run along the X-axis as opposed to Riba al-Nasi’ah facility, which runs parallel to it. Throughout this article, Riba al-Nasi’ah is assumed to compose a real return, i.e., an excess over inflation and Qardh Hasan to compose a zero real return, i.e., a nominal return equal to anticipated inflation (\(\pi\)). In deflationary environment where \(\pi < 0\), a Qardh Hasan facility would return less than the nominal amount. This assumption is inferred from the Quranic verse which states:

“Deal not unjustly and ye shall not be dealt with unjustly” (Quran 2:279).

It is not crucial to the subsequent analysis. Therefore, those dissenting from this author’s view are free to assume \(\pi = 0\).\(^7\)\(^8\)

Bai’ Murabahah (Cost Plus): Here an Islamic bank facilitates purchase of equipment/goods for an economic agent and charges a fee for its services. The title of the goods is passed to the client sub-

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\(^6\)A Qard Hasan instrument may be for a definite maturity. However, according to some scholars, this vehicle in its classical form would embody a put option for the lender (although Fiqh experts would not like to label it as an option and prefer to consider the same as an inherent right for the lender to ask for repayment). Similarly, one may think of alternative forms of Murabahah, Mudharabah, and Musharakah instruments (discussed subsequently) with or without features relating to call/put options, limited or unlimited liability, maturity, option or forward commitment of either party to sell its stake in a phased manner, etc. The facilities in their classical forms display a fairly wide range of characterizations.

\(^7\)Indexation of debt security is a highly controversial issue in Islamic economics. For a terse discussion on the subject see Hasanuz Zaman (1985).

\(^8\)Al-Omar and Abdel-Haq (1996) state that some schools in Islam allow lenders to charge service fees to compensate them for the administration of the loan. These charges are not proportional to the amount or the term of the loan or inflation. An example of an institution which adheres to this practice is the Islamic Development Bank.
Incremental Payoffs

Real Interest Payment

0

Future Value of Project

- Q

Figure 1. Riba Al-Nasi'ah (Conventional Debt) Contract

ject to a lien on it. This is removed on final payment to the Islamic bank. This contract is a very contentious issue with some denominations in Islam, which have classified it as borderline ribawi (Cizakca, 1996). However, other notable scholars in Islam such as Ibn Al-Arabi, Al-Qurtabi, and Al-Jassas have allowed it based on their commentaries of the Quran (Ismail, 1992b). Khoja (1995) also provides the daleel (religious endorsement) for the legality of this vehicle. The two conflicting views on the legality of Murabahah are reconciled by Rahman (1969), who cites instances in the life of Prophet Muhammad (PBUH) and his exemplary companions where credit sales were practiced. However, the custom was not sanctioned by later scholars as they were afraid that it would serve as a loophole to get around the restriction of riba.

Al-Omar and Abdel-Haq (1996) provide the rationale for the additional compensation to the Islamic banks based on (i) providing a needed service in locating/buying goods from a vendor for a cash-strapped agent in the economy, (ii) cost incurred by the banks in servicing their clientele, and (iii) exposing the bank to risks of the transaction explained as follows: The buyer (client) may refuse to take delivery of the goods thereby leaving the financing entity (bank)
stuck with unnecessary goods. The financial intermediary may also encounter other expenses such as storage costs and hence be exposed to damage in storage or transit. Furthermore, the client could default and the financial intermediary would have to expend resources to recoup its capital. The mark-up in the Murabahah facility is thus designed to compensate the financial intermediary for these three reasons.\footnote{There is another potential trap in the structuring of this instrument one needs to be aware of, namely the issue of delayed payments of clients. If there is a penalty involved or a mark-up over an already existing mark-up, then this would degenerate to riba al-nasi’ah. Badawi (1997) is of the view that the bank should impose a fine on the procrastinating wealthy. However, this fine should be donated to charity.}

**Bai’ Bithaman Ajil (BBA)/ Bai’ Muajjal** (deferred installment sale) is similar to Bai’ Murabahah. The main distinction here is that it is the seller or an Islamic bank who allows the deferment in payment by installment to facilitate the asset purchase. He again charges a higher fee in lieu of the service provided, expenses incurred, and exposure to risk.

From the perspective of modern finance, a Murabahah facility is equivalent to an asset-backed risky loan. If the capital markets are perfect and all agents in the economy have equal access to information, then competition between Islamic banks and conventional (ribawi) banks would result in Murabahah having the same expected return as that of ribawi loans. That is,

\[
E[\hat{r}_{\text{Murabahah}}] = i
\]

This can be further elaborated as follows. Assuming a two period framework, the Islamic bank anticipates to be paid in full when the economy is good and suffers a loss when it is bad. Let \( \hat{r}_g \) be the return in the good states of the economy, i.e., in states at or above the critical state \( s^*_c \). Below this critical state \( s^*_c \), the Islamic bank is subject to risk of loss. This can be priced in the following equivalent way with ribawi loans:

Thus, due to integration of global capital markets, Islamic banks are

\[
\int_{s_c}^{s^*_c} \hat{r}(s) \text{d}s + \int_{s^*_c}^{\infty} \hat{r}(s) \text{d}s = i
\]
\[ \Rightarrow \int_{s_0}^{s_c^*} [\tilde{r}_g - \tilde{k}(s)] \delta s + \int_{s_c^*}^{s_0} \tilde{r}_g \delta s = i, \text{where } \tilde{k}(s) \text{ is the stochastic loss parameter depending on the state of the economy } s. \]

Combining \( \tilde{r}_g \) in both integrands, the above equation is further simplified as follows.

\[ \tilde{r}_g - \int_{s_c^*}^{s_c^*} \tilde{k}(s) \delta s = i \]

\[ \Rightarrow \tilde{r}_g = i + \int_{s_0}^{s_c^*} \tilde{k}(s) \delta s = i + DP, \text{ where DP is the default premium} \quad (1a) \]

constrained to price their loans on an ex-ante basis as a function of the prevailing interest rates, as lamented by Thomas R. (1995) and Siddiqi (1995). However, their realized return would not be like that of conventional ribawi banks which can go after the other assets of the borrower and impose ribawi penalty for delayed payment. Islamic banks are exposed to cash-flow risks which can easily erode the capital base of the depositors of Islamic banks.10

A way around this dilemma as suggested by Siddiqi (1995) is to price loans based on a profit margin which does not covary with the ribawi market rate. This author agrees with Siddiqi (1995) and adds that if Islamic banks aim for a higher productivity (through optimal use of resources such as technology) they can easily underprice ribawi banks and become market leaders determining the benchmark for the allocation of funds.

**Ijara** (Operating Lease): Here again the Islamic financial intermediary facilitates the use of equipment or a productive asset to a client. The Islamic bank buys the asset and rents it to the client. The client is thus able to pay for the services of the asset from its operating income and is able to avoid an excessive initial capital outlay. The client is the lessee and the Islamic bank is the lessor. The lessor retains title to the asset until the end of the term. The lessee continues to benefit from the asset (by making regular lease payments) until the asset fails to perform the services it was originally intend-

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10This is a major difference between Islamic banks and conventional banks. There can be no guarantee of profit or even of depositors' capital. Conventional (ribawi) banks, on the other hand, guarantee depositors' capital and promise a fixed return. Any negative deviation from promised liability is absorbed by the conventional bank's capital (equity).
ed to do. The lease contract remains in force even if the asset is damaged. The asset can be sold to the lessee at the end of the lease term. However, the price of it cannot be predetermined at the beginning of the lease term.

In Lease-Purchase (Financial Lease) agreements (Ijara wal-'Iqtina) the lessor designs the lease payment to contain a portion of the price of the asset.11 The title of the asset can be transferred at the end of the lease as a gift (Khoja, 1995; Al-Omar and Abdel-Haq, 1996).

The rationale for leasing offered by Schallheim (1994) is that it provides benefits such as “tax savings, flexibility of lease contracts, savings in financial transaction, information costs, valuable options in lease contracts and risk-sharing opportunities with leasing.”12 Financial economists conceptualize leasing as a financial contract akin to that of an asset-backed risky loan. This is because leases require the lessee to make a set of fixed payments similar to a loan contract. In the worst case of default, the lessor can file a law suit against the lessee to repossess the asset and impose penalties for any possible deficiencies. In fact there is a plethora of articles in the leasing literature which discuss the extent to which leases supplement or substitute for the debt capacity of a firm (Schallheim, 1994). The basic model for pricing a lease is modified from the well-known Myers, Dill, and Bautista (1976) model. It describes the mathematical relationship between expected after-tax return on a lease and equivalent loan amount in terms of sum of after-tax lease payments, depreciation tax shelter, and expected salvage value, offset by after-tax operating cost savings such as maintenance, insurance, etc., provided by the lessor. That is, in the simple case of a single lease payment,

\[ [1 + E(\tilde{r})(1-\tau)]Q = L(1-\tau) + D\tau + E(\tilde{S}T_{g}) - O(1-\tau) \]  

Since Islamic banks as lessors provide the operating services to the lessee, one can combine the lease payment and operating service costs (after tax) and rewrite equation (2) as follows.

\[ L (1-\tau) - O(1-\tau) = L'(1-\tau) \]  

Financial economists conceptualize leasing as a financial contract akin to that of an asset-backed risky loan.

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11 In the basic Ijara (operating lease) agreement, the lease payment typically incorporate the depreciation of asset over the term of the lease.

12 Another good source for the determinants of corporate leasing policy is Smith and Wakeman (1985).
Thus equation (2) can be reformulated as follows.

\[ Q[1 + E(\bar{r})(1 - \tau)] = L'(1 - \tau) + D\tau + E(Slūg) \quad (2b) \]

\[ \Rightarrow \bar{r} = E(\bar{r}) = \frac{[L'(1 - \tau) + D\tau - Q]}{Q(1 - \tau)} + \frac{E(Slūg)}{Q(1 - \tau)} \quad (2c) \]

Although leasing firms price the asset rental payments on an ex-ante basis based on a *ribawi* benchmark such as London Interbank Offer Rate (LIBOR) (Thomas R., 1995; Siddiqi, 1995), they may not realize their expected return as the lessee may not pay on time and the lessor may not realize the anticipated salvage value—a wild card in the estimation process. Lease, McConnell, and Schallheim (1990) offer an excellent study of the divergence between realized rates of return and contractual (ex-ante) yields based on prevailing interest rates. Islamic banks are in fact exposed to cash flow risks analogous to those faced in their deferred installment sales.

In the context of project finance both *Murabahah* and *Ijara* can be illustrated by Figure 2. The cost of goods financed or asset leased is \( Q \). The bank breaks even at \( s_1 \). However, the incremental payoff in terms of a profit is only realized after a critical level \( s_c \), below which the Islamic bank receives a negative discounted return. The transaction is not *ribawi* as the incremental payoff to the Islamic bank is not fixed but is random and dependent on several factors including the success of the project.

**Islamic Income/Revenue Bond** is technically a hybrid vehicle intended for extremely risk-averse or income-oriented investors. This instrument does not guarantee any fixed *ribawi* rate of return. Instead, it promises to pay a certain fraction of the net operating income (NOI) from operations, which may be capped at a certain maximum level. It also promises the senior most claim position to the investor at the time of liquidation, when the principal (preferably in real terms) is returned to the investor.\(^{13}\) This bond is an extension of the classical *Mudharabah* instrument used historically in Islam, as explained in the next subsection. A variant of this vehicle termed *Esham* shares was offered by the Ottoman Empire following the 1768–1774 Russo–Turkish war. A similar version designated as Revenue Participating Scheme (GOS) was reintroduced by the Ozal government in Turkey in the mid 1980s (Cizakca, 1996). A related mech-

\(^{13}\)In contrast to a *ribawi* bond, the Islamic bond should preferably return the principal in real terms and not in nominal terms. If it returns the principal in nominal terms, it can be construed to be against the norms of justice or fair-dealing in Islam (Badawi, 1996).
Incremental Payoffs

![Graph showing incremental payoffs]

**Figure 2. Murabahah/Ijara Contracts**

anism is also used by the Ministry of Awqaf of the Government of Jordan as reported by El-Hennawi (1990).

**HYBRID (MUDHARABAH) SECURITIES**

Muslims are extremely nostalgic about *Mudharabah* financing as the holy Prophet Muhammad (PBUH) himself resorted to this facility as a trader. This was also selected by his eminent companions and his family members for investing surplus funds of orphans (Kahf and Khan, 1992; Siddiqi, 1985). The Prophet (PBUH) endorsed this security with the following statement narrated by his companion Sahib and recorded in Ibn Majah:

“There are three things which have the blessings of Allah (the Glorious): deferred payment sale, *al-Muqaradah* (*al-Mudharabah*)
and mixing of barley with wheat for home consumption (not for sale).” (Ibn Majah: Tijarah Chapter: 63)

Historically, Mudharabah was used in trade financing where the financier (rabb-ul-mal) supplied the funds while the trader (mudharib) provided sweat equity. Cizakca (1996) illustrates how this financing mechanism played a vital role in the economic development of the Muslim world in its glorious era and how this was adopted by Europe in the Medieval period as a partnership form known as commenda.

The Mudharabah security combines the features of both equity and Islamically allowed debt contracts (Bacha, 1995). Here the investor (rabb-ul-mall) has a junior position with respect to debt investors discussed previously. However, it has a preferred position over that of an equity investor (musharik). Figure 3 illustrates this feature showing that under the state $s_1$, the Rabb-ul-mal (financier) is not able to recoup his capital and suffers a loss. Between the states $s_1$ and $s^*_c$, he breaks even. However, beyond the critical state $s^*_c$ the Rabb-ul-mal shares in ‘$\Theta’$ fraction of the profit. In most of the Islamic Banking and Economics literature this is termed the Profit Sharing Principle (PSP). The modern finance equivalent to this instrument is a participating preferred stock with no contractually promised interest. Inclusion of participation clauses in a financial contract mitigates the stockholder-bondholder conflict, enhances the value of a

Figure 3. Mudharabah Contract (Assuming No Other Senior Debt Contract)
Incremental Payoffs

Profit

Future Value of Project

\( S_1 \)

- \( Q \)

Incremental Payoffs

\( \theta (\text{Call Premium}) \)

Future Value of Project

\( S^*_c \)

\( S_2 \)

- \( C \)

Murabahah (Cost Plus)

Figure 3A. Synthesizing a Mudharabah Security
project and serves to control the under-investment problem (Haugen and Senbet, 1981, 1987; Green, 1984; Schnabel, 1993).

**Proposition:** A Mudharabah facility can be synthesized as a combination of a Murabahah (cost-plus) facility and fractional ($\theta$) shares of Al-Ikhtiyarat (European Call Option) such that the profit from the Murabahah in the good states of the economy offsets the Call Price.\(^{14}\)

**Proof:** Figure 3A illustrates the fact that the Mudharabah is forged when the cost (premium) of fractional shares ($\theta$) of a European Call Option equals profit from a Murabahah in the good states of the economy. That is, $\theta C_0 = Q(\bar{r}_g)$. Thus, the profit sharing parameter $\theta$ can be endogenously determined as follows.

$$\theta = \frac{(Q_{Murabahah})_0[\bar{r}_g\text{ Murabahah}]}{C_0},$$

where $C_0$ is the initial price of the European Call Option.

**Equation (3)**

Equation (3) has some crucial ramifications. It defines the profit sharing ratio as a simple ratio, which dynamically varies with the return from the Murabahah as well as with the Option Premium. This result is contrary to some researchers of Islamic economics who have assumed it to be invariant to the specificity of the project and across time.

Mathematically, the initial and maturity value of the Mudharabah can be priced as follows.

$$\begin{align*}
(P_{Mudharabah})_0 &= (Q_{Murabahah})_0 \\
(P_{Mudharabah})_T &= (Q_{Murabahah})_0 + \theta C_T \\
&= (Q_{Murabahah})_0 + \theta C_T \\
&= (Q_{Murabahah})_0[1 - \bar{k}(s)] \\
&\forall s \leq s^*_c, \text{ where } C_T \\
&\text{is the maturity value of the Option and } \bar{k}(s) \text{ is the stochastic loss parameter.}
\end{align*}$$

**Equation (3a)**

The value of the European Call Option in Equation (3) can be resolved under an investment setting similar to that of the classical

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\(^{14}\)Kamali (1997) extensively discusses the controversy surrounding Al-Ikhtiyarat (Options) based on the Islamic perspective. He is of the view that options do not involve Gharrar and can be traded for a premium (price).
Black-Scholes (1973) model. The basic methodology of this model stems from a Qardh Hasan (risk-free) hedge composed of one share of stock of a firm involved in the project and fractional shares of a Call Option sold against it (Ritchken, 1987). The derivation assumes the following:\(^\text{15}\)

1. Frictionless capital markets with no transaction costs or taxes and with information simultaneously available to all agents in the economy without any costs.
2. No restrictions on borrowing assets based on Qardh Hasan.
3. Continuous asset trading in the firm with a single project, with all firm prices following continuous and stationary stochastic processes.
4. Nonstochastic risk-free rate on the Qardh-Hasan vehicle equal to the inflation rate \((\pi)\).
5. No dividends.

The results from the Black and Scholes (1973) analysis adapted to the current situation is given below.

\[
\tilde{C}_T = \text{Final Value of a European Call Option} \\
= \max[0, (\tilde{V}_T - V_0)], \text{ where } V_0 \text{ and } \tilde{V}_T(s) \text{ are the initial and maturity values of the project, respectively. The value of the project is composed of the value of its assets and net operating income. That is, } V_j = P_j + \text{NOI}_j \forall j
\]

\[
C_0 = \text{Initial Value of a European Call Option} \\
= V_0 N(d_1) - X N(d_2), \text{ where } X \text{ is the exercise price of the option in real terms, i.e., taking expected inflation } (\pi) \text{ under consideration; } T \text{ is the time to maturity, and } N(*) \text{ is the cumulative normal density function.}
\]

Since the option involves sharing of appreciation in the real value of the firm, \(X = V_0\). The above formula simplifies to:

\(^\text{15}\)The numerical illustration at the end of this article presents a simpler version of option pricing model based on a binomial distribution as opposed to the Black-Scholes (1973) model based on a continuous Log-Normal Distribution. The two option pricing models are related (Ritchken, 1987).
\[ C_0 = V_0[N(d_1) - N(d_2)], \] where \[ d_1 = \frac{(\pi + 0.5(\sigma^2))T}{\sigma(T^{0.5})} = \frac{\pi}{\sigma} + \frac{\sigma}{2}, \] when \( T = 1 \) and \[ d_2 = d_1 - \sigma T^{0.5} = \frac{\pi}{\sigma} - \frac{\sigma}{2}, \] when \( T = 1. \]

### EQUITY SECURITIES—AL-MUSHARAKAH

*Musharakah*, derived from the Arabic term *Shirkah*, implies partnership in a venture. The classical Islamic business literature lists many forms of business enterprises (Chapra, 1986; Siddiqi, 1985). The one implied by this study is *Shirkah al-'Inan*, i.e., partnership with unequal contribution of capital and with different profit and loss sharing ratios. The conventional financial instrument which is closest to the above Islamic concept is equity as it connotes ownership (control) of an enterprise. This offers the highest potential reward at the expense of highest possible risk. This is because an equity owner is a residual claimant in any venture. He hopes to gain the most under favorable economic conditions but loses the most under unfavorable conditions. This is the reason why this contract is referred to as the Profit and Loss Sharing (PLS) contract. Figure 4 illustrates this fact by showing that the equity owner suffers a loss below the future critical state \( s^* \), breaks even at \( s^* \), and profits from the venture if the future state is better than \( s^* \). In a situation where the project is funded by both *Mudharabah* and *Musharakah*, the musharik retains only \((1 - \theta)\) percent of the profits.

The mathematical pricing of the *Musharakah* facility is described by the Zero-Beta version of the Capital Asset Pricing Model (Black, 1972; Lintner, 1969) as follows.

\[ E(\tilde{r}_S) = E(\tilde{r}_Z) + \beta_{SM}[E(\tilde{r}_M) - E(\tilde{r}_Z)], \]

\[ [E(\tilde{r}_M) - E(\tilde{r}_Z)] > 0, \text{ and} \]

\[ \text{Cov}(\tilde{r}_M, \tilde{r}_Z) = 0, \text{ where } \tilde{r}_S, \tilde{r}_Z \text{ and } \tilde{r}_M \text{ imply the stochastic returns on the stock, the zero-beta portfolio and the market portfolio, respectively.} \]

This formula could be further simplified if the zero-beta asset is defined as a *Qardh-Hasan* asset, which is to be paid off in full under all states of the economy in real terms. With this interpretation, equity in an Islamic economy can be priced as:
Incremental Payoffs

Future Value of Project

(1-\(\Theta\)) = Residual Profit Sharing Ratio

Figure 4. Musharakah Contract (Assuming a Senior Mudharabah Contract)
\[ E(\bar{r}_S) = \pi + \beta_{SM}[E(\bar{r}_M) - \pi], \text{ and} \]

\[ [E(\bar{r}_M) - \pi] > 0, \text{ where } \pi \text{ is the expected rate of inflation. } \tag{4a} \]

**TWO-DIMENSIONAL MALLEABILITY OF THE MUDHARABAH FACILITY**

*Mudharabah* is a very versatile instrument in two separate dimensions described subsequently in combination with the *Musharakah* vehicle.

**Case VA. Adapting the Mudharabah Instrument to Cater to the Financial Objective of the Clientele**

The classical *Mudharabah* facility can be explained as a growth instrument where a trader would reinvest the proceeds of all trade until the termination of the contract where the principal amount would be returned to the *rabb-ul-mal* and the excess (profits) would be shared according to the agreed ex-ante profit sharing ratio. In case of a loss, the balance or residual would accrue to the *rabb-ul-mal*. In the context of modern finance this device can imply sharing or allocating a fraction of income or the appreciation of a project (business venture). Instead of terminating a *Mudharabah* contract at an inopportune moment, it is preferable to sell it in the secondary market. The profit sharing parameters of income and appreciation can be reset by a custodian bank (in consultation with the management of the company which represents the *musharik*, i.e., equity investor) according to the changes in (i) economic conditions, (ii) risk of the project (business ventures), and (iii) clienteles of this vehicle. Thus, in modern times one can financially engineer a *Mudharabah* instrument to suit the financial goals of the investor in several forms described as follows.

*Income Mudharabah*: Here the investor shares in the income or revenue of the project only and is paid back the principal (preferably in real terms) at the termination of the contract. This is similar to the income/revenue bond described in the previous subsection.

*Growth and Income Mudharabah*: Here the investor shares in a fraction of NOI as well as the appreciation of the project (venture). Both the fractional parameters of income and appreciation need not be equal. In terms of conventional finance, this can be interpreted as a participating bond.
Growth Mudharabah: This is similar to the classical Mudharabah where it is assumed that the project (business venture) has an optimal policy of paying no dividends. All the earnings are retained in the business and distributed at the termination of the business. This is rather cumbersome and restrictive in contemporary times as incorporated businesses or ventures are assumed to have an infinite life.

Case VB. Adapting the Mudharabah Facility to Cater to the Risk Profile of the Clientele

This is perhaps one of the most contentious issues confronting the Islamic financial engineer. It can be addressed as follows: Is it Islamically feasible for the rabb-ul-mal (of the Mudharabah) to transfer part of the risk of the venture to the musharik in exchange for a lower profit sharing ratio and vice versa? This can be accomplished by partially combining the Mudharabah discussed previously with a protective Put Option such that the premium of the Put is offset by the Call premium in the Mudharabah resulting in a lower profit sharing ratio. When the buyer of the protective Put is the rabb-ul-mal (and the seller obviously is the musharik), the situation resembles Figure 5. Here the musharik guarantees to bear the risk of part (λ) of the funds of the rabb-ul-mal in return for a higher participating ratio (1 - θ + φ). Thus, for future states of the economy below s₁, the rabb-ul-mal’s payoff schedule shrinks inward parallel to the original schedule and for states above s₁, it rotates clockwise. For the musharik the risk now increases by the amount guaranteed (λ) for states below s₁ and for states above s₁, the payoffs (in terms of the profit sharing ratio) increase by φ. In the limit as λ approaches Q (nominal risk becomes zero for the rabb-ul-mal), f approaches the limit φ* s.t. θ* → (θ - φ*) and the Mudharabah instrument tends to become a fully hedged vehicle. It should be noted that even though risk is reduced in nominal terms, there is still residual risk due to inflation. Finally, the opposite scenario can also be configured when the musharik transfers part of the risk of equity to the rabb-ul-mal in return for a lower profit sharing ratio. The payoff schedule of the rabb-ul-mal below s₁ would expand outwards and above s₁ rotate anticlockwise. The crucial debatable fiqhi (Islamic jurisprudence) issue is as follows: To what extent is this risk-return trade-off allowed in Islam? Note that the reduction in risk is arrived at by creating a portfolio of a Mudharabah facility and a Takaful (insurance) policy in the form of a Put Option, both of which are permissible in Islam (Kahf and Khan, 1992; Kamali, 1997). This is an Ijtihadi issue left for the Ulema of the Fiqh Academies to resolve.
**OPTIMAL FINANCIAL STRUCTURE OF A PROJECT**

Optimization depends on a whole host of factors such as the return-risk characteristics of the project (which are dependent on the ex-ante expected economic conditions), the risk profile of the ultimate security holders, and their time horizon. The return-risk characteristics of the project can be adjusted by proper coordination of the various professionals involved in the project. For example, prior to undertaking the project, the Islamic banker needs to be sure that it is technically feasible. Furthermore, marketing management can mitigate the risk of the project by getting pre-commitments from potential clients or customers, and operations management can ensure that the project is completed on time. The optimal design of its financial structure de-
pends on the risk-profile of potential agents. Islamic bankers have to design incentive-compatible contracts which comply with the Islamic principles as discussed earlier. This is done by modeling each potential investor's objective function and solving for Pareto-optimal contracts which maximize the welfare of the agents in the economy. A numerical example given below presents one possible solution for a project without a rigorous and detailed general equilibrium analysis. Variants of the Mudharabah facility are utilized as they can be easily securitized without violating the strictures of Bai' al-dayn bi al-dayn (sale of one set of debt security with another).

**Numerical Illustration**

Consider a two-year project, which follows a double binomial distribution such that the liquidation value of the assets and NOI follow a geometric Wiener process, where the appreciation parameter ($u$) is given by 1.1, depreciation parameter ($d$) by 0.975 and expected inflation ($\pi$) by 0.0375. The project's characteristic is described in Figure 6. It encompasses tangible assets and net operating income (NOI) the base values of which are $910 million and $60 million, re-

![Figure 6. Asset Price/NOI (Options) Realizations in a Two-Period Binomial Model](image-url)
spectively. However, the cost of undertaking the complete project is negotiated at $910 million. The desired expected internal rate of return (IRR) of the project equals 10 percent. Investigate whether this project should be accepted and if accepted, discuss an Islamically feasible mode of financing.

Solution

Table 1 systematically delineates the solution to this problem. Section A of Table 1 first estimates the probability of appreciation ($p$) as given by the following formula in Ritchken (1987).

$$
\theta = \frac{(R - d)}{(u - d)}
$$

(5)

The variables in this equation have the following interpretation: $R$ is one plus the return on a Qardh-Hasan (risk-free) asset, $u$ is the appreciation parameter, and $d$ is the depreciation parameter of the two-period binomial distribution.

Since $R = (1 + \pi) = 1.0375$, $u = 1.1$, and $d = 0.975$, this implies $p = 50$ percent = 0.5

The next step is to evaluate the expected overall internal rate of return of the project. This is determined to be 10.49 percent which is higher than the desired internal rate of return of 10 percent. Therefore, the project should be accepted as it improves the economic welfare of the investors.16

The subsequent step is to price the options on the growth ($C_0$) and income ($C'_0$) of the project respectively as depicted in Section B using the pricing formula for the two-period binomial distribution given by Ritchken (1987) as follows.

$$
C_0 = \frac{[\theta^2C_{22} + 2\theta(1-\theta)C_{21} + (1-\theta)^2C_{20}]}{R^2}
$$

(5a)

$$
C'_0 = \frac{[\theta^2C'_{22} + 2\theta(1-\theta)C'_{21} + (1-\theta)^2C'_{20}]}{R^2}
$$

(5b)

16Note the pricing for various Islamic instruments derived earlier assumed competitive markets in contrast to the current example.
Table 1. Simulation Results

The model is solved for the endogenous variables such as the probability of appreciation \( \theta \), expected IRR of the project, price of the options on the project's asset \( (C_0^o) \) and NOI \( (C_0'^o) \). These are used to derive the profit-sharing parameters of the Pure Income Bondholder and Participating Bondholder. The exogenous parameters assumed are as follows: (i) \( u \) (geometric growth factor) = 1.1, \( d \) (geometric decline factor) = 0.975; (ii) rate of inflation \( (\pi) \) = 3.75 percent; (iii) cost of undertaking the project = 910 million; (iv) net operating income generated by project in the first and second years are anticipated to follow a geometric random walk from a base level of $60 million; (v) Income bondholder contributes 50 percent of project value whereas the Participating bondholder contributes 25 percent according to the following desired allocation: 10 percent Income component and 15 percent growth component; (vi) the expected returns of Murabahah's assigned to the income and growth components are 5 percent and 6 percent, respectively; and (vii) the growth component is shared between the Participating bondholder and the Equity holder.

Section A (Evaluation of Expected IRR of Project)

Evaluation of Probability of Appreciation

\[ \theta = 50\% \]

Evaluation of Expected IRR of Project

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>( CF_0^o(\text{mil.}) )</th>
<th>( CF_1^o(\text{Mil.}) )</th>
<th>( CF_2^o(\text{Mil.}) )</th>
<th>IRR %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good ( (u,u) )</td>
<td>-910.00</td>
<td>66.00</td>
<td>1173.70</td>
<td>17.2527</td>
</tr>
<tr>
<td>M1 ( (u,d) )</td>
<td>-910.00</td>
<td>66.00</td>
<td>1040.33</td>
<td>10.6091</td>
</tr>
<tr>
<td>M2 ( (d,u) )</td>
<td>-910.00</td>
<td>58.50</td>
<td>1040.33</td>
<td>10.1838</td>
</tr>
<tr>
<td>Dab ( (d,d) )</td>
<td>-910.00</td>
<td>58.50</td>
<td>922.11</td>
<td>3.9286</td>
</tr>
</tbody>
</table>

\( \text{Exp(IRR)} = 10.4935\% \).

Section B (Pricing of Project Asset and NOI Options)

Pricing of Asset Option

\[
\begin{align*}
P_{22} &= $1101.1000 \text{ Mil.} \\
P_{21} &= $975.9750 \\
P_{20} &= $865.0688 \text{ Mil.} \\
C_{22} &= $121.5703 \text{ Mil.} \\
C_{21} &= 0 \\
C_{20} &= 0 \\
\text{Res}_{22} &= $295.9266 \text{ Mil.} \\
\text{Res}_{21} &= $241.3277 \text{ Mil.} \\
\text{Res}_{20} &= $130.4215 \text{ Mil.}
\end{align*}
\]

\( C_0 = $28.23523 \text{ Mil.} \)

Pricing of NOI Option

\[
\begin{align*}
NOI_{11} &= $66 \text{ Mil.} \\
NOI_{10} &= $58.5 \text{ Mil.} \\
NOI_{22} &= $72.6 \text{ Mil.} \\
NOI_{21} &= $64.35 \text{ Mil.} \\
NOI_{20} &= $57.0375 \text{ Mil.} \\
C_{22}' &= $72.6 \text{ Mil.} \\
C_{21}' &= $64.35 \text{ Mil.} \\
C_{20}' &= $57.0375 \text{ Mil.}
\end{align*}
\]

\( C_0' = $60 \text{ Mil.} \)

Section C (Design of Securities)

<table>
<thead>
<tr>
<th>Stakeholder</th>
<th>Contribution (%( V^o_0 ))</th>
<th>Exp. Return on Murabahah (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Pure Income Bondholder</td>
<td>50% (Income Component)</td>
<td>5</td>
</tr>
</tbody>
</table>

*continued*
Section C (Design of Securities) Continued

<table>
<thead>
<tr>
<th>Stakeholder</th>
<th>Contribution (%V₀)</th>
<th>Exp. Return on Murabahah (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Participating Bondholder</td>
<td>10% (Income Component)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>15% (Growth Component)</td>
<td>6</td>
</tr>
<tr>
<td>3 Equity</td>
<td>Residual</td>
<td>Residual</td>
</tr>
</tbody>
</table>

C1: Murabahah income expected by income bondholder = $22.75 Mil.
    Percentage of income allocated to income bondholder = 37.9167 percent.
C2: Murabahah income expected by participating bondholder = $4.55 Mil.
    Percentage of income allocated to participating bondholder = 7.5833 percent.
C3: Murabahah growth expected by participating bondholder = $16.38 Mil.
    Percentage of growth allocated to participating bondholder = 68.0126 percent.
C4: Residual income accruing to equity holder = 54.5 percent.
    Residual growth accruing to equity holder = 41.9874 percent.

Section D (Determination of Expected IRR of Each Financier)


<table>
<thead>
<tr>
<th>State of Economy</th>
<th>CF₀(Mil.)</th>
<th>CF₁(Mil.)</th>
<th>CF₂(Mil.)</th>
<th>IRR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good(u,u)</td>
<td>-455</td>
<td>25.0250</td>
<td>517.2923</td>
<td>9.4113</td>
</tr>
<tr>
<td>M1(u,d)</td>
<td>-455</td>
<td>25.0250</td>
<td>514.1642</td>
<td>9.0885</td>
</tr>
<tr>
<td>M2(d,u)</td>
<td>-455</td>
<td>22.1813</td>
<td>514.1642</td>
<td>8.7684</td>
</tr>
<tr>
<td>Bad(d,d)</td>
<td>-455</td>
<td>22.1813</td>
<td>511.3916</td>
<td>8.4814</td>
</tr>
</tbody>
</table>

Exp(IRR) = 8.9374 percent.

D2. Evaluation of Exp. IRR of Participating Bondholder (Growth and Income Rabb-ul-Mal):

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>CF₀(Mil.)</th>
<th>CF₁(Mil.)</th>
<th>CF₂(Mil.)</th>
<th>IRR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good(u,u)</td>
<td>-227.5</td>
<td>5.0050</td>
<td>320.9141</td>
<td>19.8743</td>
</tr>
<tr>
<td>M1(u,d)</td>
<td>-227.5</td>
<td>5.0050</td>
<td>249.7623</td>
<td>5.8844</td>
</tr>
<tr>
<td>M2(d,u)</td>
<td>-227.5</td>
<td>4.4363</td>
<td>249.7623</td>
<td>5.7582</td>
</tr>
<tr>
<td>Bad(d,d)</td>
<td>-227.5</td>
<td>4.4363</td>
<td>249.2078</td>
<td>5.6418</td>
</tr>
</tbody>
</table>

Exp(IRR) = 9.2897 percent.

D3. Evaluation of Expected IRR of Equity holder (Musharik):

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>CF₀(Mil.)</th>
<th>CF₁(Mil.)</th>
<th>CF₂(Mil.)</th>
<th>IRR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good(u,u)</td>
<td>-227.5</td>
<td>35.9700</td>
<td>335.4936</td>
<td>29.5997</td>
</tr>
<tr>
<td>M1(u,d)</td>
<td>-227.5</td>
<td>35.9700</td>
<td>276.3985</td>
<td>18.4129</td>
</tr>
<tr>
<td>M2(d,u)</td>
<td>-227.5</td>
<td>31.8825</td>
<td>276.3985</td>
<td>17.4539</td>
</tr>
<tr>
<td>Bad(d,d)</td>
<td>-227.5</td>
<td>31.8825</td>
<td>161.5069</td>
<td>-8.4452</td>
</tr>
</tbody>
</table>

Exp(IRR) = 14.2553 percent.
The various parameters are explained as follows.

\[
\begin{align*}
C_{22} &= \text{Max.}((uP_0 - X), 0), \quad C'_{22} = \text{Max.}((u\text{NOI}_0 - X'), 0) \\
C_{21} &= \text{Max.}((udP_0 - X), 0), \quad C'_{21} = \text{Max.}((ud\text{NOI}_0 - X'), 0) \\
C_{20} &= \text{Max.}((dP_0 - X), 0), \quad C'_{20} = \text{Max.}((d\text{NOI}_0 - X'), 0) \\
X &= R^2P_0 \\
X' &= 0.
\end{align*}
\]

Thus, the value of the project’s growth \((C'_0)\) and income \((C'^{'}_0)\) options are evaluated as $28.235 million and $60 million, respectively.

Section C (Table 1) assumes that the Income Bondholder (Pure Income Rabb-ul-Mal) and Participating Bondholder (Growth and Income Rabb-ul-Mal) contribute 50 percent and 25 percent of the initial value of the project, respectively. However, the Participating Bondholder allocates 10 percent and 15 percent of his funds to the income and growth components, respectively. Assuming the expected returns on equivalent Murabahah financing on the income and growth components to be 5 percent and 6 percent, the allocation ratios of both financiers are appraised in Sub-Sections C1 and C2 respectively using Equation (3). Thus, the Pure Income Bondholder (and Participating Bondholder) are designated 37.92 percent (7.58 percent) of the project’s income and 0 percent (58.01 percent) of the project’s growth, respectively. The residual component of the income and growth accrues to the Equity holder (musharik).

Finally, Section D (Table 1) evaluates the rate of return across different future states of the economy for each stakeholder of the firm. The income bondholder’s compensation varies from 8.48 percent to 9.41 percent with an expected value of 8.9 percent. The Participating Bondholder’s return ranges from 5.64 percent to 19.87 percent with an expected value of 9.29 percent, while that of the Equity holder’s lies from -8.45 percent to 29.6 percent with an expected value of 14.26 percent.

It should be observed that the expected return and risk increase from Income Bondholders to the Equity investors. In the poor state of the economy the Equity investor fares the worst, followed by the Participating Bondholder, followed by the Income Bondholder. In the good state of the economy the situation is reversed. The financial system design is an incentive-compatible system as the Equity owner is required to work hard to earn the maximum benefit. It should be noted here that if the situation ex-post turns out to be worse than anticipated, then all the participants of the project would suffer economic losses. However, the party to lose the most would be the Musharik (Equity) followed by the Growth and Income Rabb-ul-Mal (Participating Bondholder) followed by the Pure Income Rabb-ul-Mal (Income Bondholder).
CONCLUSION

This study interpreted and modeled Islamic project finance instruments in the language of a conventional banker. A Mudharabah is envisioned as a combination of a Murabahah facility and an Option. This leads to an endogenous estimation of the profit-sharing parameter. The Mudharabah is pliable along two separate dimensions, i.e., the financial objective and risk tolerance of the investor. The incentive compatible financial design of a project was illustrated in this article by a numerical example using a combination of Mudharabah and Musharakah facilities. The security design aspect of this study has a wide array of application ranging from the financial engineering of Islamic money market instruments to that of exotic hybrid vehicles catering to the goals and risk profile of its clientele.

About the Author

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